Representing Temporal Interval Using Conceptual Graphs

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1 ABSTRACT

This paper shows how conceptual graphs can be used to represent relationships among temporal intervals. It shows how Allen's 13 relations, 20 end point relations, and 16 cover relations can be mapped to 12 base relations. A transitivity table is given for these base relations and a rule given for propagating temporal constraints through a conceptual graph of conjunctive temporal relations.. This paper also shows how the contexts of conceptual graphs can be used to help control the combinatorial explosion common with such propagation algorithms.

2 BACKGROUND

In his classic paper "Maintaining Knowledge about Temporal Intervals"[1] James Allen showed that only thirteen relations are needed to represent all possible temporal relationships between two time intervals. These are defined in Figure 1 where x and y denote temporal intervals. The first and second columns specify the full name and short hand name given by Allen. The third column gives a pictorial definition of the relations. In the pictures, the left end of the interval indicates the start and the right end the finish of the interval. Shown on the right is the equivalent representation in terms of the conceptual graphs defined by Sowa.[2]

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[1] "Maintaining Knowledge about Temporal Intervals," James F, Alien, Comm. ACM, Vol. 26, No. 11, pp832-843, Nov. 1983.

[2] [Conceptual Structures: Information in Mind and Machine, John F. Sowa, Addison-Wesley, Reading, MA, 1984.

x RELATION y	KEY	PICTURE
x BEFORE y	B	
x MEETS y	M	
x OVERLAPS y	O	
x FINISHED BY y	Fi	
x CONTAINS y	C	
x STARTS y	S	
x EQUALS y	E	
x STARTED BY y	Si	
x DURING y	D	
x FINISHES y	F	
x OVERLAPPED BY	Oi	
x MET BY y	Mi	
x AFTER y	A	

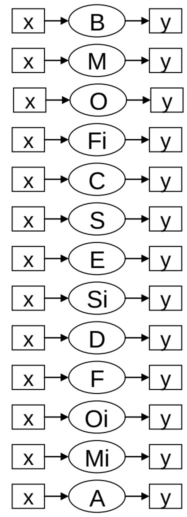

Figure 1. Allen's Relations on Intervals

Allen also showed that these temporal relations are closed under transitivity which, consequently, allows one to reason about networks of time intervals connected by these interval relations. He did this by specifying a transitivity table. The rows and columns are indexed by his 13 interval relations and the intersections specify the disjunction of his relations that is the transitive closure value for that pair of relations. (Section 4 has more on transitivity.)

In "A Common-Sense Theory of Time"[3] Allen provided a sounder mathematical foundation for time interval relationships. He showed that all of the thirteen relations could be expressed in terms of just the MEETS relation as shown in Figure 2.

[3] "A Common Sense Theory of Time," J. F. Alien & P. J. Hayes, Proc. of IJCAI-85, pp528-531, 1985.

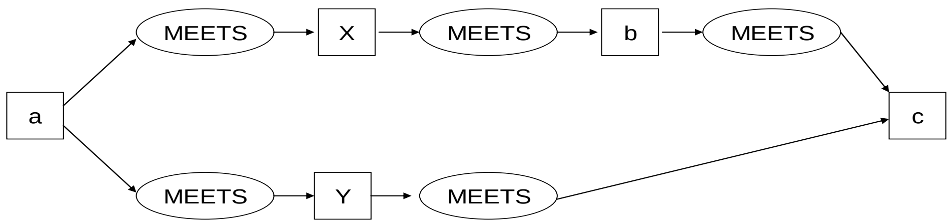

Figure 2. Example STARTS Relation in terms of MEETS Relation.

Besides these definitions, six axioms about the MEETS relation were needed to provide a well-defined logic system. Figure 3 illustrates one of the axioms using conceptual structures notation. (The dotted lines indicate that the concepts[4] refer to the same set of intervals.) Axiom Ml states that if the end points of two intervals I and J both meet an end point of a third interval K, then any interval m that one interval meets, the other also meets.

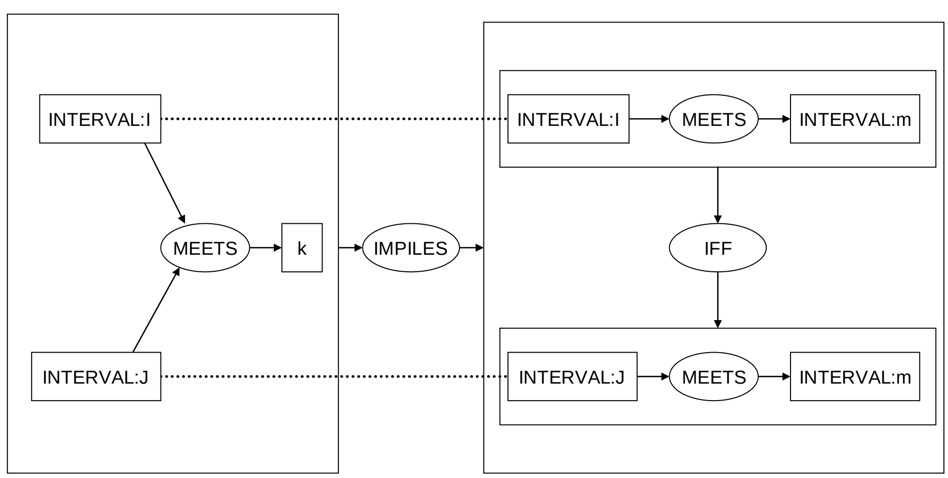

Figure 3. Axiom Ml for Uniqueness of "Meeting Places"

[4] The original paper used ovoids, but that was confusing so they were changed to the normal rectangles.

Lastly, Allen extended his theories and showed that they applied to both discrete or continuous time or, for that matter, to any fully ordered discrete or continuous relation. This is very important because it means that the representations and software are applicable to a broad set of domains. Examples are distance and probability intervals. As shown in Figure 1, conceptual graphs can represent temporal intervals in a very natural way. The next section will show how the relation definition capability of conceptual graphs can be used to define a wide variety of relations in terms of base relations or the MEETS relation, as desired. This ability, along with the ability to define axioms as shown in Figure 3, means conceptual graphs can be used to define complete canons for temporal logic.

Allen's relations completely define the relationship of the end points of both intervals with respect to each other, in many cases one does not know these relationships completely. Knowing less about the relationship between two intervals requires a disjunction of Allen's relations. Because conceptual graphs are based on conjunction and negation, disjunctions are not easily expressed by them.

Look at the pictorial column of Figure 1 and note that the STARTS, EQUALS, and STARTED-BY relations all restrict the two intervals to start at the same time. This situation is denoted by the SWS (starts when starts) relation. Figure 4 show how messy a definition is when expressing disjunction in terms of conceptual graphs . (The situation is actually more complex than shown because one has to use contexts and negation to define the OR relation.)

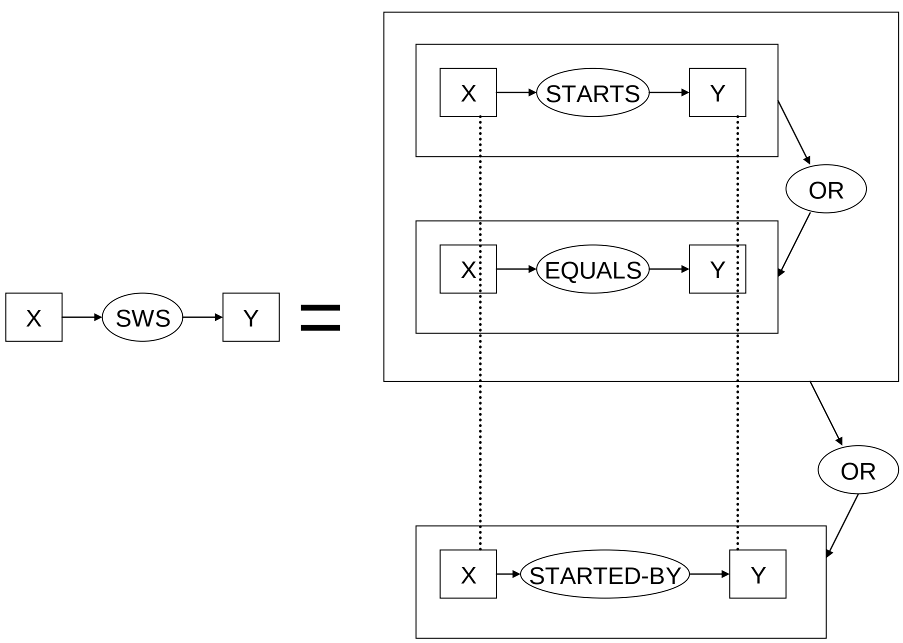

Figure 4. Example Endpoint Relation as a Disjunction.

3 SOLVING THE DISJUNCTION PROBLEM

In "Endpoint Relations on Temporal Intervals"[5] Matuszek and his colleagues solve this problem by defining a set of twelve endpoint relations like SWS. More specific relationships between intervals are expressed as conjunctions of endpoint relations. This is shown in Figure 5 which gives a definition of Allen's STARTS relation in terms of the conjunction of two endpoint relations.

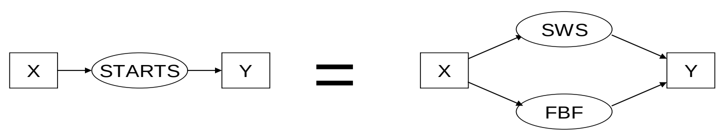

Figure 5. Example Allen's Relation as a Conjunction.

Figure 6 gives a more interesting example. The top part uses the pictorial notation to define some relationships among intervals having to do with two activities, gardening and making dinner. Since the relationships among the intervals are completely known, -the conceptual graph describing some of these relationships using Allen's intervals is very natural and compact. However, the . equivalent graph in tents of endpoint relations is not much more complex because of the naturalness of conjunction in conceptual structure theory.

[5] "Endpoint Relations on Temporal Intervals," D. Matuszek, T. Finin, R. Fritzson, & C. Overton, Unisys, Paoli Res. Ctr., Tech. Rpt. PRC_LBS_8810, June 1988.

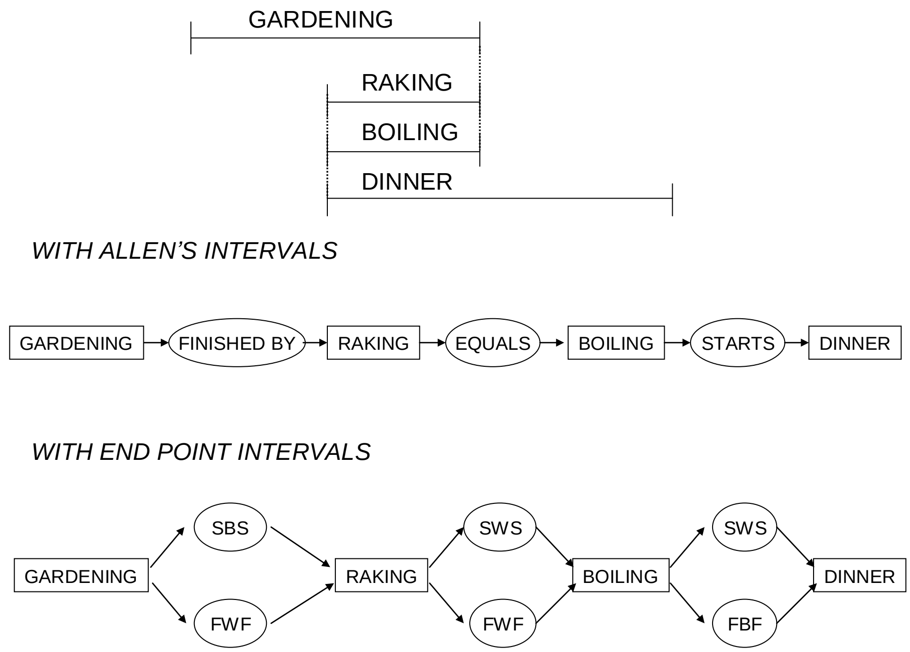

Figure 6. Example of Intervals & Corresponding Networks

4 BASE RELATIONS

When trying to combine Allen's and Matuszek's work, another family of relations was discovered. We called them "cover" relations because they specify the relationship of one end of an interval to the inside of a second interval. "Inside" needed better definition as to whether the ends of the second interval defined an open or closed Interval. To include a broad range of cases and make The names of the relations more intuitive, we adopted the mathematical notation for distinguishing open and closed intervals. Thus [ ] is the closed interval, includes the end points, ( ) is the open interval, [ ) is open on the right, and ( ] is open on the left.

The four cases are compounded by two additional degrees of freedom. One is whether the start or finish of the first interval is being covered. The other is whether the first or second interval is doing the covering. The net result is 16 different cover relations. We also applied the open verses closed distinction to Matuszek's endpoint relations which expanded their number to 20. In theory a transitivity table with 49 (13+16+20) relations on a side is possible, but it is very difficult to work with. To solve this problem, we identified 12 base relations which are shown in Figure 7. Because the base relations only specify constraints on one end of each relation, the other end is often not constrained. This is indicated by the dotted line showing a range of values still possible for the free end of the X relation.

<table>
<tr><th>x RELATION y</th><th>PICTURE
|------y------|</th></tr>
<tr><td>x S)S y</td><td>|--x--…</td></tr>
<tr><td>x S]S y</td><td>|……....|--x--…</td></tr>
<tr><td>x S=S y</td><td>|--x--…</td></tr>
<tr><td></td><td></td></tr>
<tr><td>x S)F y</td><td>|……....|--x--…</td></tr>
<tr><td>x S]F y</td><td>|…………………|--x--…</td></tr>
<tr><td>x S=F y</td><td>|--x--…</td></tr>
<tr><td></td><td></td></tr>
<tr><td>x F)S y</td><td>…--x--|</td></tr>
<tr><td>x F]S y</td><td>…--X--|…|</td></tr>
<tr><td>x F=S y</td><td>…---x-----|</td></tr>
<tr><td></td><td></td></tr>
<tr><td>x F)F y</td><td>…--x--|……|</td></tr>
<tr><td>x F]F y</td><td>…--x--|…………….|</td></tr>
<tr><td>x F=F y</td><td>….--x--|</td></tr>
</table>

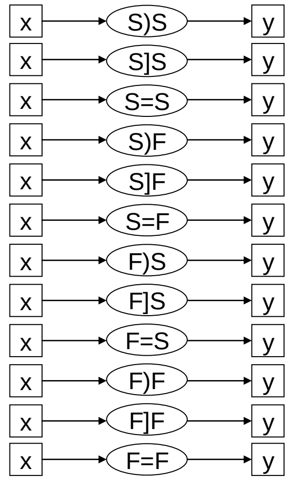

Figure 7. Base Relations on Intervals

For example, S)S means that the x relation starts before the y relation starts and F]S means that x finishes at or before the y relation starts.
We used these 12 base relations to define the remaining 37 relations (12+37«49) in terms of them or a conjunction of them. These definitions are given in Table 1. In it the columns are divided into 3 groups of 3 columns each. Each group of 3 columns defines one of the sets of relations. Within each group, the “Rel” column identifies the relation being defined from interval x to interval y. The second and third columns give base relations which are conjoined to give the relation being defined. (Note that the base relation in the third column is from y to x.)

ENDPOINT			COVER			ALLEN ‘ S		
Rel	xRy &	yRx	Rel	xRy &	yRx	Rel	xRy &	yRx
S)S =	S)S		S[] =	S]F	S]S	B =	F)S	
S]S =	S]S		[]S =	S]S	S]F	M =	F-S	
S=S =	S=S		S[) =	S)F	S]S	0 =	S)S	of
S[S =		S]S	[)S =	S]S	S)F	Fi =	S)S	F-F
S(S =		S)S	S(] =	S]F	S)S	C =	S)S	F)F
S)F =	S)F		(]S =	S)S	S]F	S =	F)F	S-S
S]F =	S]F		S() =	S)F	S)S	E =	S=S	F=F
S=F =	S=F		()S =	S)S	S)F	Si =	S=S	F)F
S[F =		F]S	F[] =	F]F	S]F	D =	F)F	S)S
S(F =		F)S	[]F =	S]F	F]F	F =	F=F	S)S
F)S =	F)S		F[) =	F)F	S]F	Oi =	S()	F)F
F]S =	F]S		[)F =	S]F	F)F	Mi =	S=F	
F=S =	F=S		F(] =	F]F	S)F	A =		F)S
F[S =		S]F	(]F =	S)F	F]F			
F{S =		S)F	F() =	F)F	S)F			
F)F =	F)F		()F =	S)F	F)F			
F]F =	F]F							
F=F =	F=F							
F[F =		F]F						
F(F =		F)F						

The problem has now been divided into two parts, defining the transitive closure over the 12 base relations and defining a rule for the case where there are conjunctions of base relations.

5 TRANSITIVE CLOSURE

First a short explanation. Look at Figure 6 again and determine the relationship between GARDENING and DINNER for yourself. By looking at the pictorial diagram, you can easily see that the two intervals overlap. However, the question is, how can we get computers using conceptual graphs to come to the same conclusion?

Figure 8 names the nodes and gives an answer graph. For transitive closure to hold, it must be possible, given any two relations, such as R4 between I1 and I3, and R3 between I3 and I4 to infer R5, the relationship of I1 to I4 directly. In the case of intervals, the transitive closure is given by a table indexed by relations. For R4 = FINISHED-BY and R3 = STARTS, the entry in the table would be OVERLAPS, labeled R5.

In our approach, we want to work with the 12 base relations instead of Allen's relation because other relations can be expressed as a conjunction of them. We prefer them to Matuszek's because they cover more cases. The transitive closure for them is given in tabular form in Table 2.

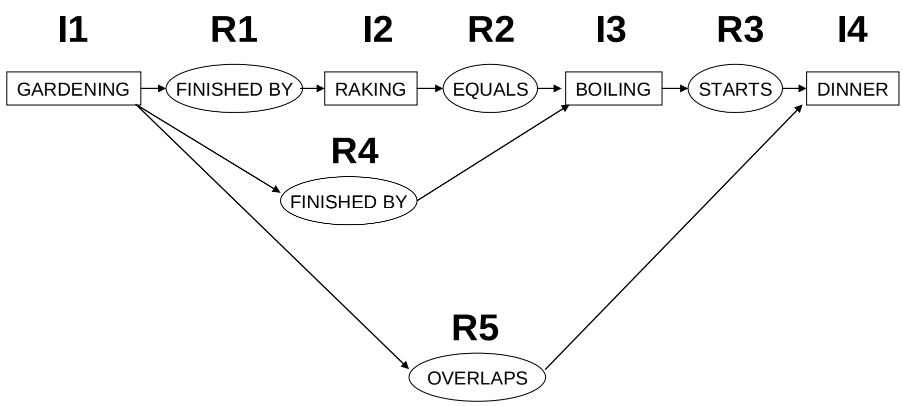

Figure 8. Interval Network with Labeled Nodes.

rel	s)s	s]s	s=s	s(s	s[s	s)f	s]f	s=f	s(f	s[f	f)s	f]s	f=s	f(s	f[s	f)f	f]f	f=f	f(f	f[f
s)s	s)s	s)s	s)s	-	-	s)f	s)s	s)f	-	-	s)s	s)s	s)s	-	-	s)f	s)f	s)f	-	-
s]s	s)s	s]s	s]s	-	-	s)f	s]f	s]f	-	-	s)s	s)s	s)s	-	-	s)f	s)f	s)f	-	-
s=s	s)s	s)s	s=s	s(s	s(s	s)f	s)f	s=f	s(f	s(f	s)s	s)s	s)s	-	-	s)f	s)f	s)f	-	-
s(s	-	-	s(s	s(s	s(s	-	-	s(f	s(f	s(f	-	-	-	-	-	-	-	-	-	-
s[s	-	-	s[s	s(s	s[s	-	-	s[f	s(f	s[f	-	-	-	-	-	-	-	-	-	-
s)f	-	-	-	-	-	-	-	-	-	-	s)s	s)s	s)s	-	-	s)f	s)f	s)f	-	-
s]f	-	-	-	-	-	-	-	-	-	-	s)s	s]s	s]s	-	-	s)f	s]f	s]f	-	-
s=f	-	-	s(s	s(s	s(s	-	-	s(f	s(f	s(f	s)s	s)s	s=s	s(s	s(s	s)f	s)f	s=f	s(f	s(f
s(f	-	-	s(s	s(s	s(s	-	-	s(f	s(f	s(f	-	-	s(s	s(s	s(s	-	-	s(f	s(f	s(f
s[f	-	-	s[s	s(s	s(s	-	-	s(f	s(f	s(f	-	-	s[s	s(s	s[s	-	-	s[f	s(f	s[f
f)s	f)s	f)s	f)s	-	-	f)f	f)f	f)f	-	-	f)s	f)s	f)s	-	-	f)f	f)f	f)f	-	-
f]s	f)s	f]s	f]s	-	-	f)f	f]f	f]f	-	-	f)s	f)s	f)s	-	-	f)f	f)f	f)f	-	-
f=s	f)s	f)s	f=s	f)s	f)s	f)f	f)f	f=f	f(f	f(f	f)s	f)s	f)s	-	-	f)f	f)f	f)f	-	-
f(s	-	-	f(s	f(s	f(s	-	-	f(f	f(f	f(f	-	-	-	-	-	-	-	-	-	-
f[s	-	-	f[s	f(s	f[s	-	-	f[f	f(f	f[f	-	-	-	-	-	-	-	-	-	-
f)f	-	-	-	-	-	-	-	-	-	-	f)s	f)s	f)s	-	-	f)f	f)f	f)f	-	-
f]f	-	-	-	-	-	-	-	-	-	-	f)s	f]s	f]s	-	-	f)f	f]f	f]f	-	-
f=f	-	-	f(s	f(s	f(s	-	-	f(f	f(f	f(f	f)s	f)s	f=s	f(s	f(s	f)f	f)f	f=f	f(f	f(f
f(f	-	-	f(s	f(s	f(s	-	-	f(f	f(f	f(f	-	-	f(s	f(s	f(s	-	-	f(f	f(f	f(f
f[f	-	-	f(s	f(s	f(s	-	-	f(f	f(f	f(f	-	-	f[s	f(s	f[s	-	-	f[f	f(f	f[f

Table 2. Transitive Closure for Base Relations.

Since one doesn't know which order one is going to encounter the relations, the inverses of the 12 base relations are also given.

The second part of the problem is a rule for dealing with conjunctions. The FINISHED-BY relation equals S)S and F=F, and the STARTS relations equals S=S and F)F. The rule for handling conjunctions is: form the conjunction of the transitive closure of all possible pairs of base relations.

For our example, the relation of I1 to I4[6] is the conjunction of the transitive closure (using Table 2) of all possible pairs of base relations between I1 and I3, and between I3 and I4. The set between I1 and I3 is {S)S F=F} and that between I3 and I4 is (S=S F)F). The pairings and results of looking them up in Table 2 are:

PAIR		TRANSITIVE CLOSURE
S)S	S=S	S)S
S)S	F)F	S)S
F=F	S=S	F(S
F=F	F)F	F)F

The S)S relation implies the S)F relation because, by definition, all relations start before they finish. By using Table 1 backwards, it can be seen that the last two relations are the same as F(). (Note that the inverse[7] of F(S is S)F, the second entry in Table 1 for F().) Using Table 1 backwards again, with S)S and ()F, the inverse of F(), we see that the conjunction of these three is equivalent to OVERLAPS, the desired result.

[6] Some of the references to intervals were wrong in the original paper and have been corrected.
[7] By inverse here it is meant xRy to yRx, because the third columns of each set is yRx.

6 SUMMARY

This paper has combined Allen's interval relations with an extended version of Matuszek's endpoint relations and a new set of relations called cover relations. Collectively, these provide a very flexible family of 49 relations for expressing relationships among temporal or other kinds of intervals.

These relations are defined in terms of 12 base relations or conjunctions of them. These conjunctions can be represented directly in conceptual graphs as multiple relations between two intervals represented as concepts.

Networks of intervals connected by interval relations are naturally represented in conceptual graphs. The propagation of interval constraints, as interval relations are added or deleted, is accomplished by using a transitivity table over the 12 base relations and a rule to handle conjunctive cases.